Sensing of Particle Shape and Size Using Arrays of Artificial Cilia

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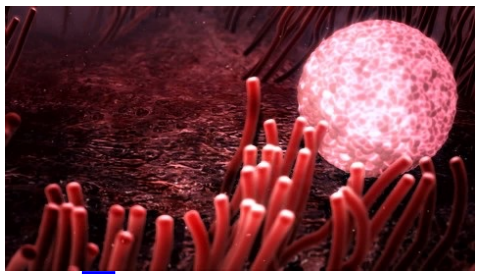
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6 June 2024

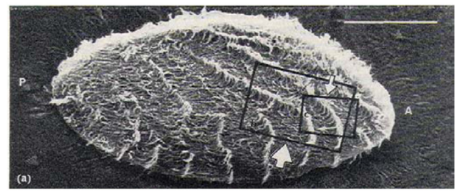
ECCOMAS

What, Why & How?

What?: Hair like organelles, Cilia are ubiquitous, found in animals and microorganisms, performing various roles



Ovum Transport (Source: Web)



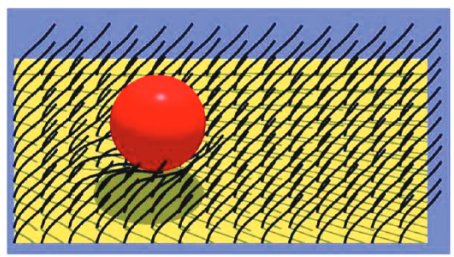
Metachronal waves on Opalina

Reproduced from (Tamm and Horridge 1970)(©2014, The Royal Society.

Why?: Understanding the functionality of cilia can lead to the development of

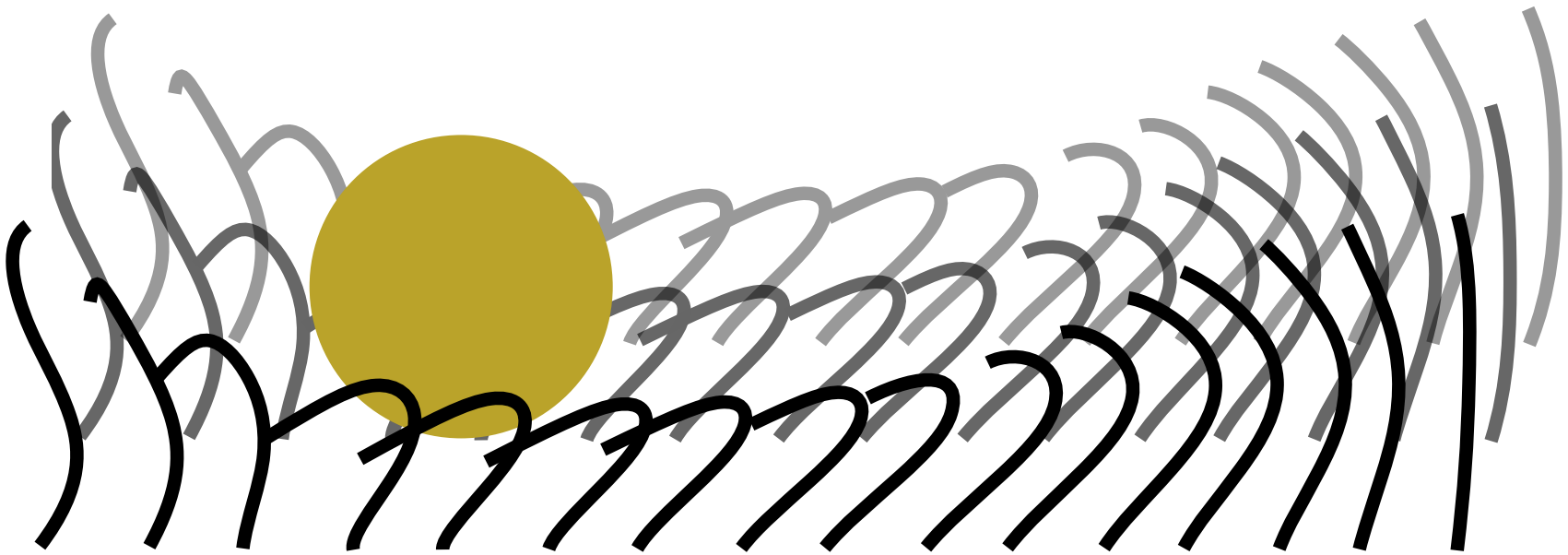
- Sensors which detect particulate flows and its characteristics
- Design of drug-delivering artificial swimmers

How?: Computational Study



Bhattacharya and Balazs 2013

Objective



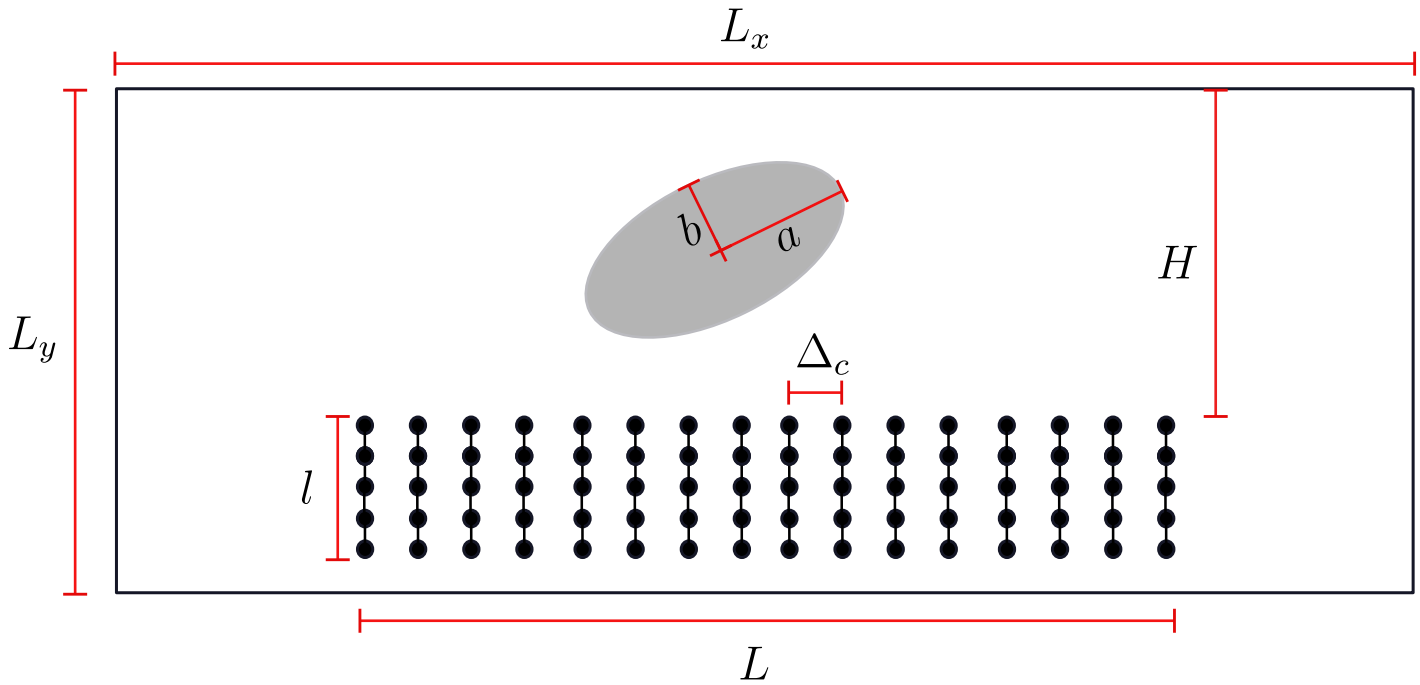
To identify the features of particles through their interactions with non-motile cilia.

Two Parts

1. Data Generation (Numerical Simulations)
2. Machine Learning

Data Generation

Numerical Simulations (Fluid-Structure Interactions)



Computational Domain

Boundary	Boundary Conditions	
	Velocity	Pressure
Top	No-Slip ($u_x = U_{Top}, u_y = 0$)	Zero-Gradient
Bottom	No-Slip ($\mathbf{u} = 0$)	Zero-Gradient
Left	Zero-Gradient	Constant (P=0)
Right	Zero-Gradient	Constant (P=0)

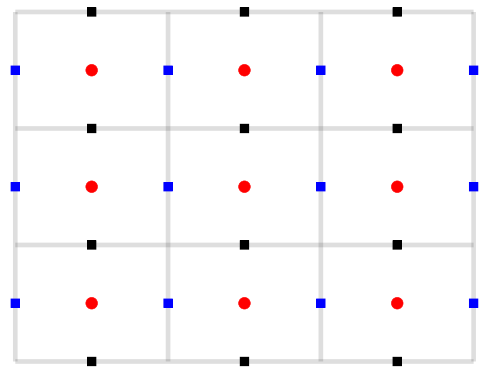
Fluid Solver

Equations

Two-dimensional, unsteady, and incompressible Navier-Stokes equations

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}^b$$

$$\nabla\cdot\mathbf{u} = 0$$



Staggered Grid

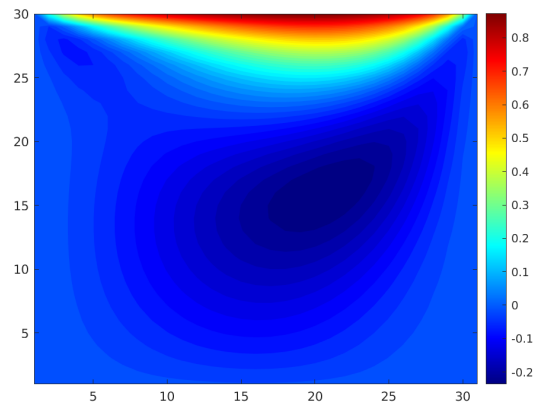
How do we calculate $\mathbf{f}^b$?

Spatial Discretization

Finite difference method (Uniform staggered grid). An upwind scheme is for the advective term.

Temporal Discretization

Explicit Euler scheme

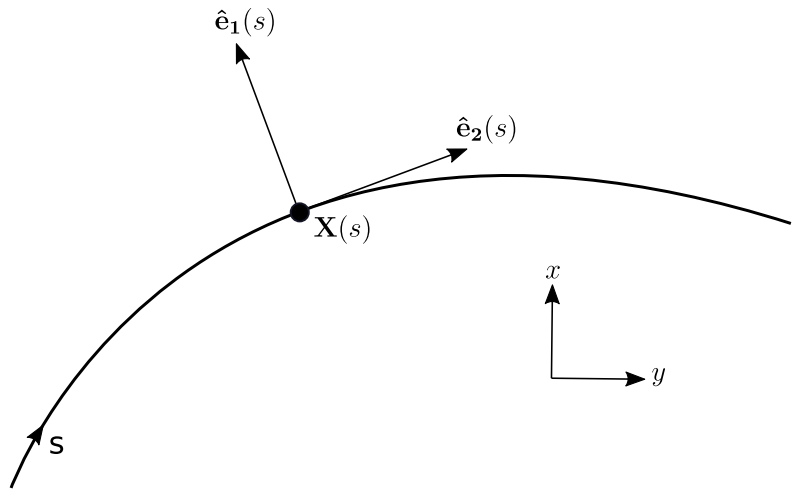


Lid driven cavity simulation

Cilia Solver

A cilium is modeled as a Kirchoff rod following the methodology prescribed in (Lim et al. 2008), The following are the Equilibrium equations across a section of the curve.

$$0 = \mathbf{f} + \frac{\partial \mathbf{F}}{\partial s} \qquad 0 = \mathbf{n} + \frac{\partial \mathbf{N}}{\partial s} + \frac{\partial \mathbf{X}}{\partial s} \times \mathbf{F}$$



$$F_1 = b\hat{\mathbf{e}}_1 \cdot \frac{\partial \mathbf{X}}{\partial s} \qquad F_2 = b\left(\hat{\mathbf{e}}_2 \cdot \frac{\partial \mathbf{X}}{\partial s} - 1\right) \qquad N = a\frac{\partial \hat{\mathbf{e}}_2}{\partial s} \cdot \hat{\mathbf{e}}_1$$

The discretization scheme provided in Lim et al. 2008 is used.

Particle Solver

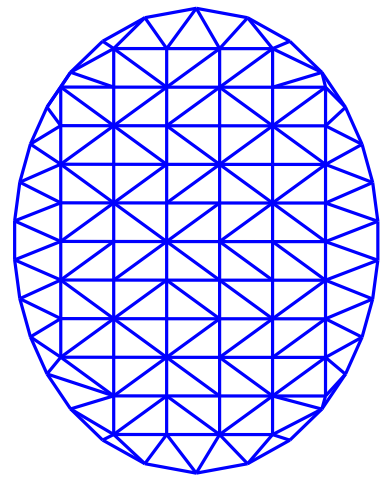
Particles are assumed massless and characterized by a neo-Hookean strain energy density function (Bhattacharya and Balazs 2013).

$$u = \frac{c_o}{2}(I_1 - 3) - c_o \ln J + \frac{1}{2}K(\ln J)^2$$

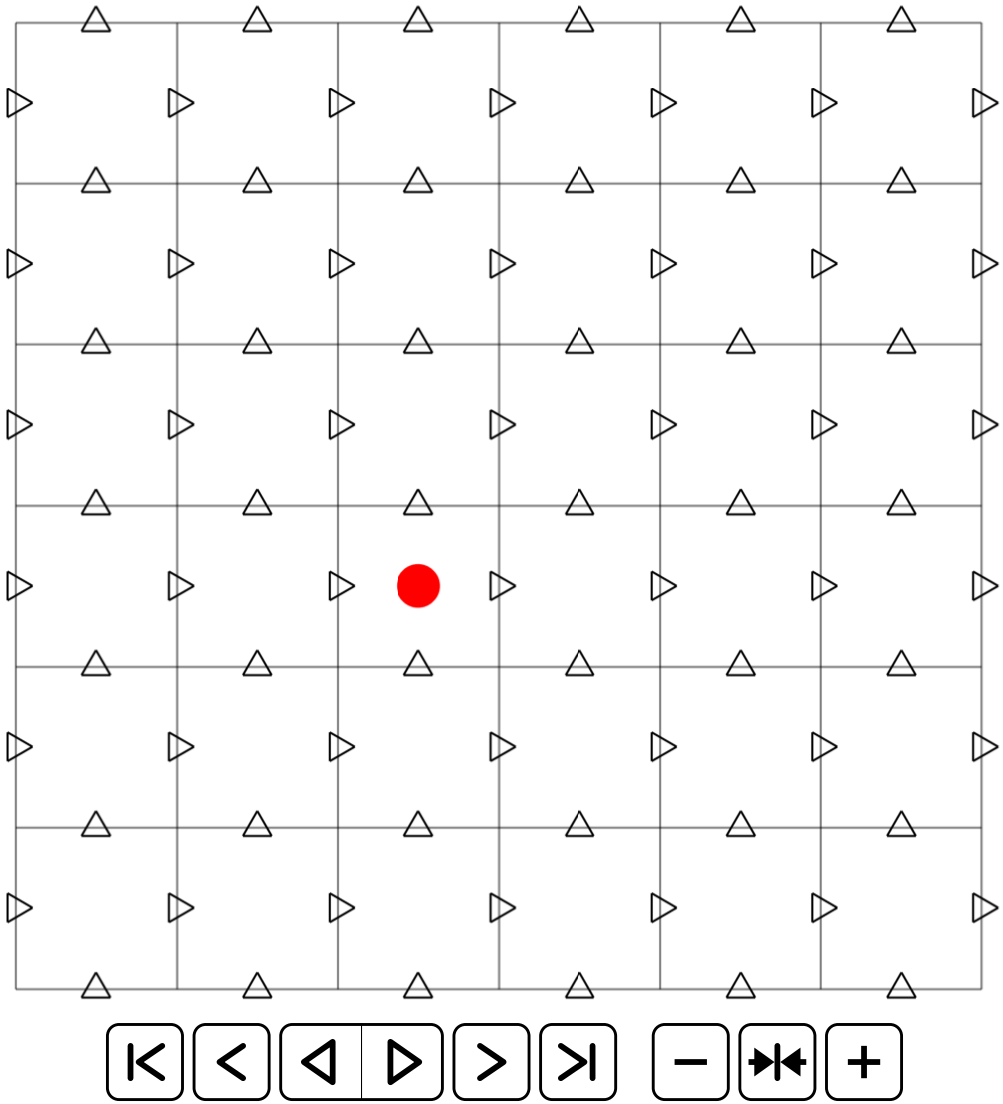
For small deformations, $E = c_o(2c_o + 3K)/(c_o + K)$ and $\nu = K/[2(c_o + K)]$ where E is the Young's modulus and ν is the Poisson's ratio of the particle

The force at each node of every triangle in the discretization is calculated as,

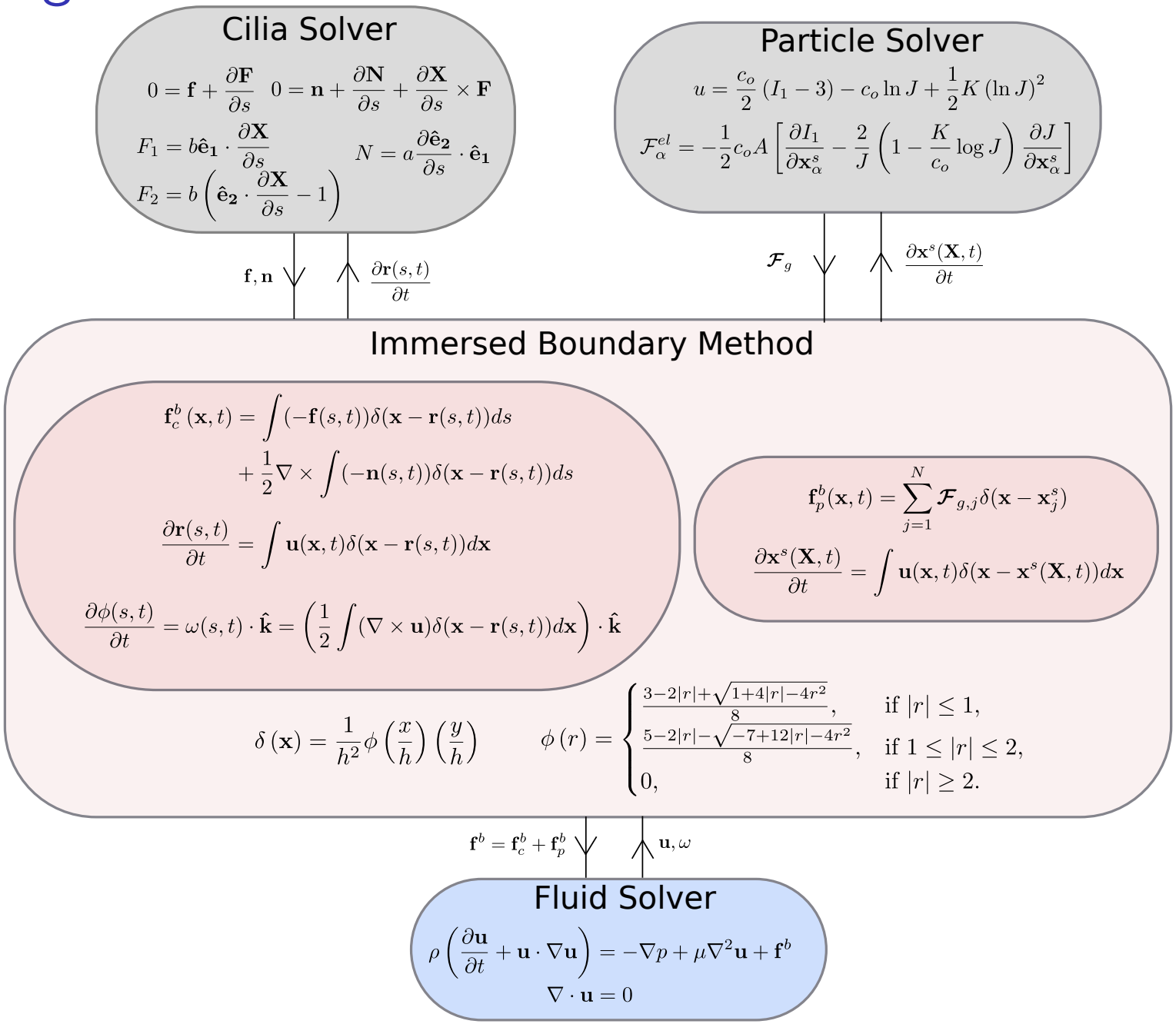
$$F_i^\alpha = -\frac{1}{2}c_o A\left[\frac{\partial I_1}{\partial x_i^\alpha} - \frac{2}{J}\left(1 - \frac{K}{c_o}\log J\right)\frac{\partial J}{\partial x_i^\alpha}\right]$$



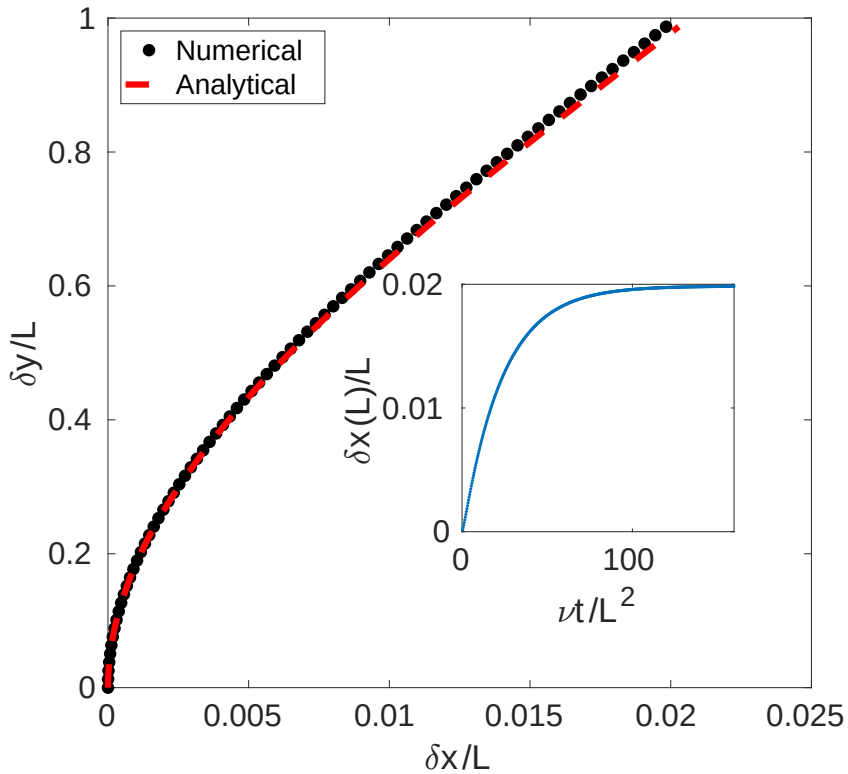
Immersed Boundary Method



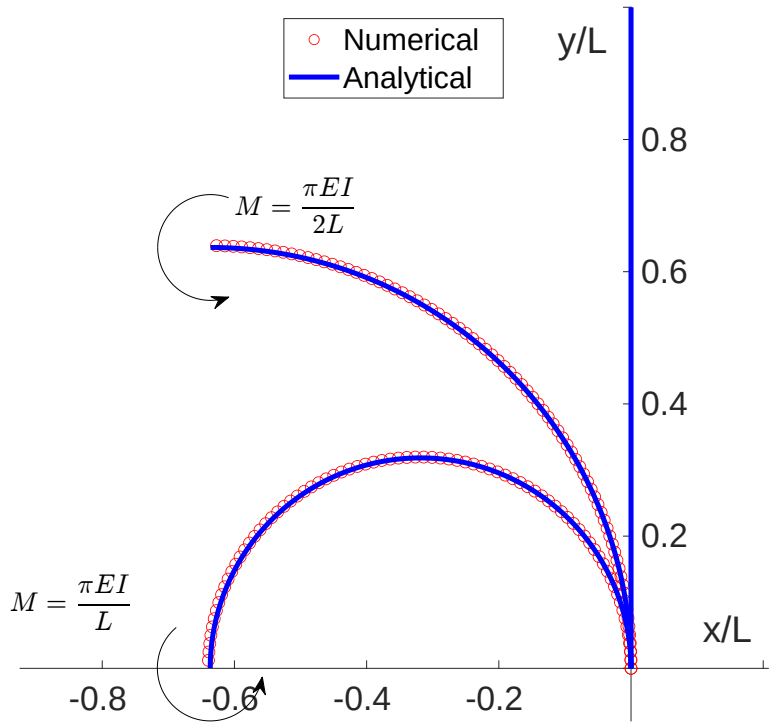
Coupling of Solvers



Validation



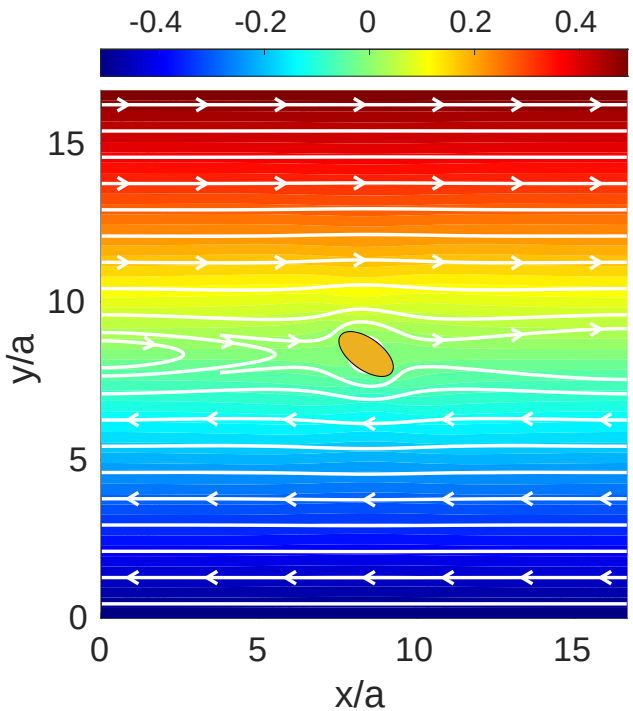
Deflection of cantilever cilium subjected to a point load it's free-end



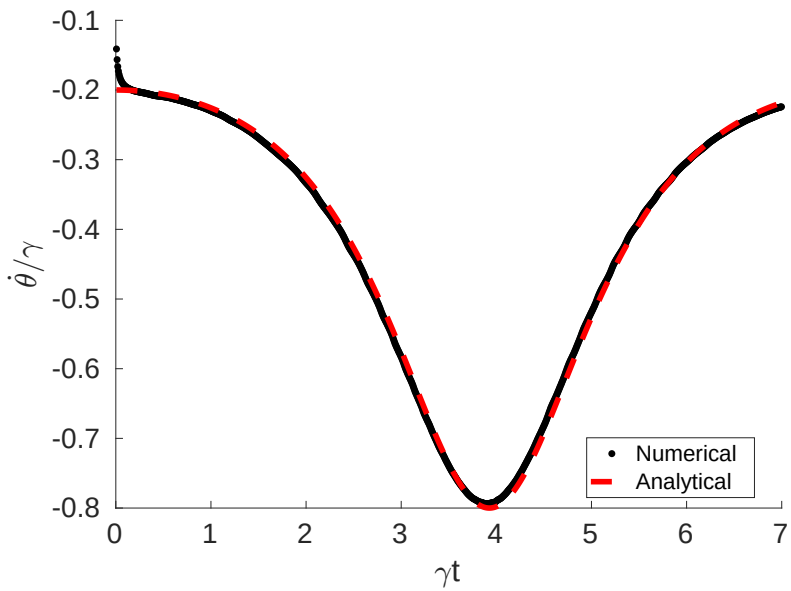
Pure bending of cantilever cilium subjected to moment at it's free-end

Validation

The angular velocity of the particle in shear flow is validated against the analytical results of Jeffery 1915.



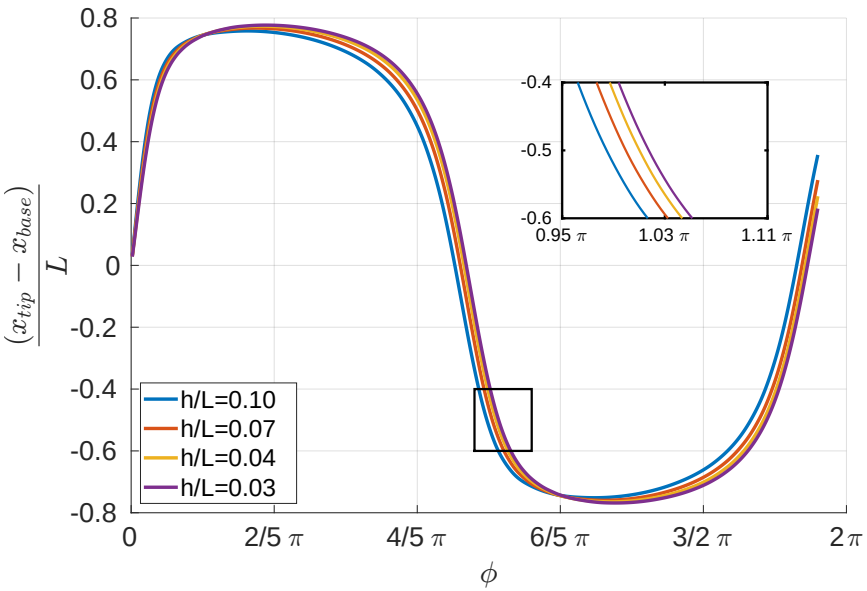
Filled isocontours of ua/ν



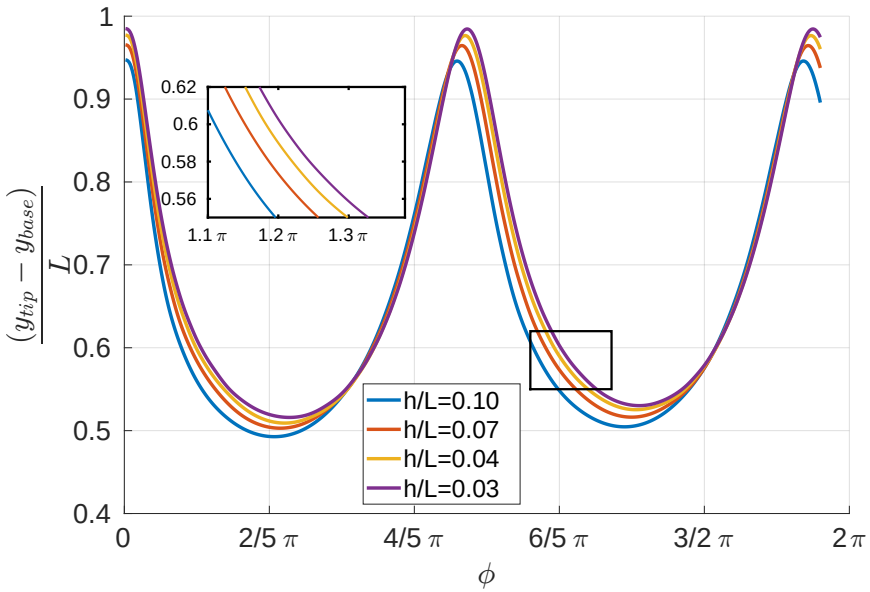
Comparison with Jeffery 1915

$$Re = U_{\mathrm{Top}} a/\nu = 0.51$$

Grid Independence

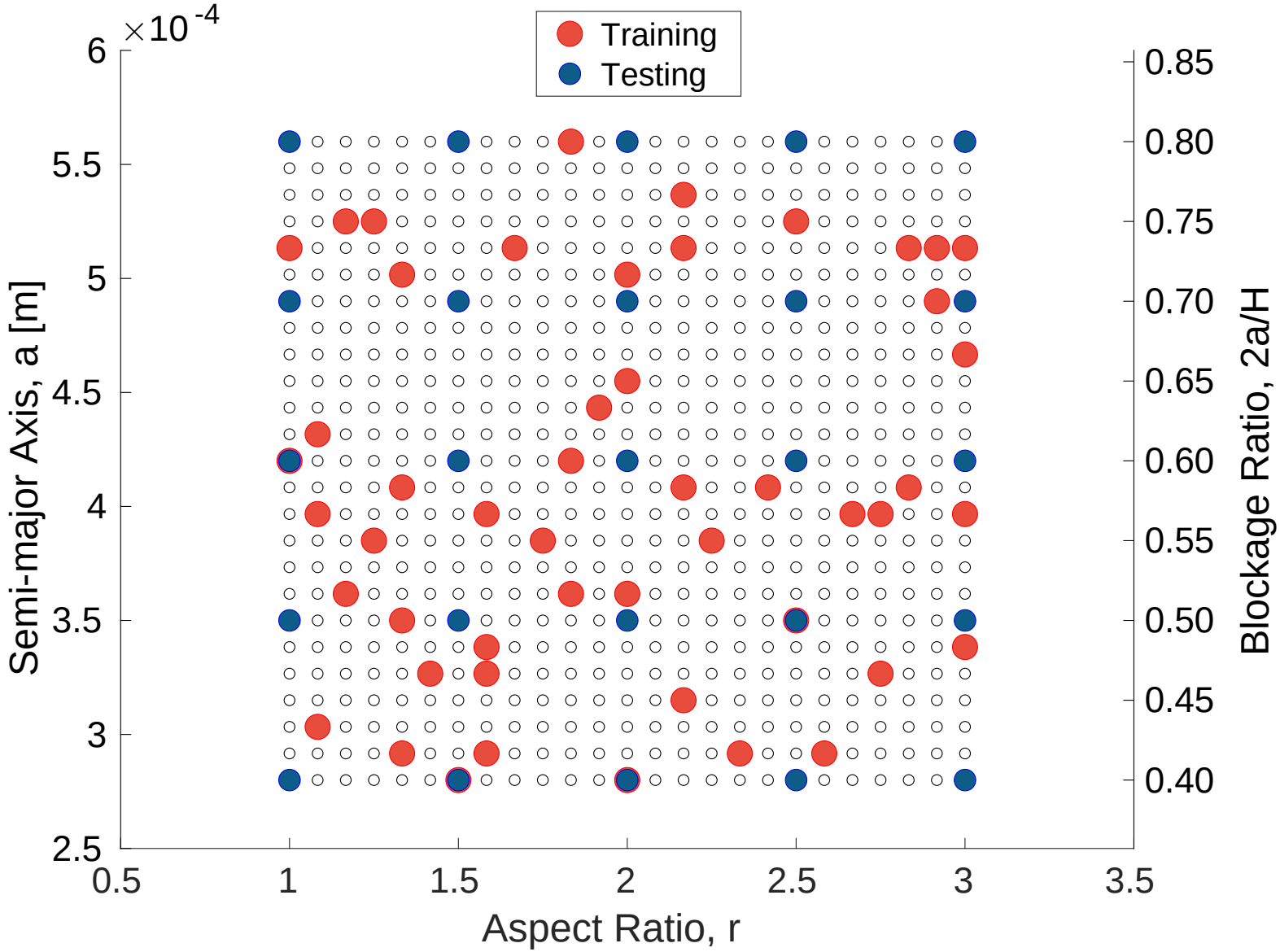


x component of cilia deflection



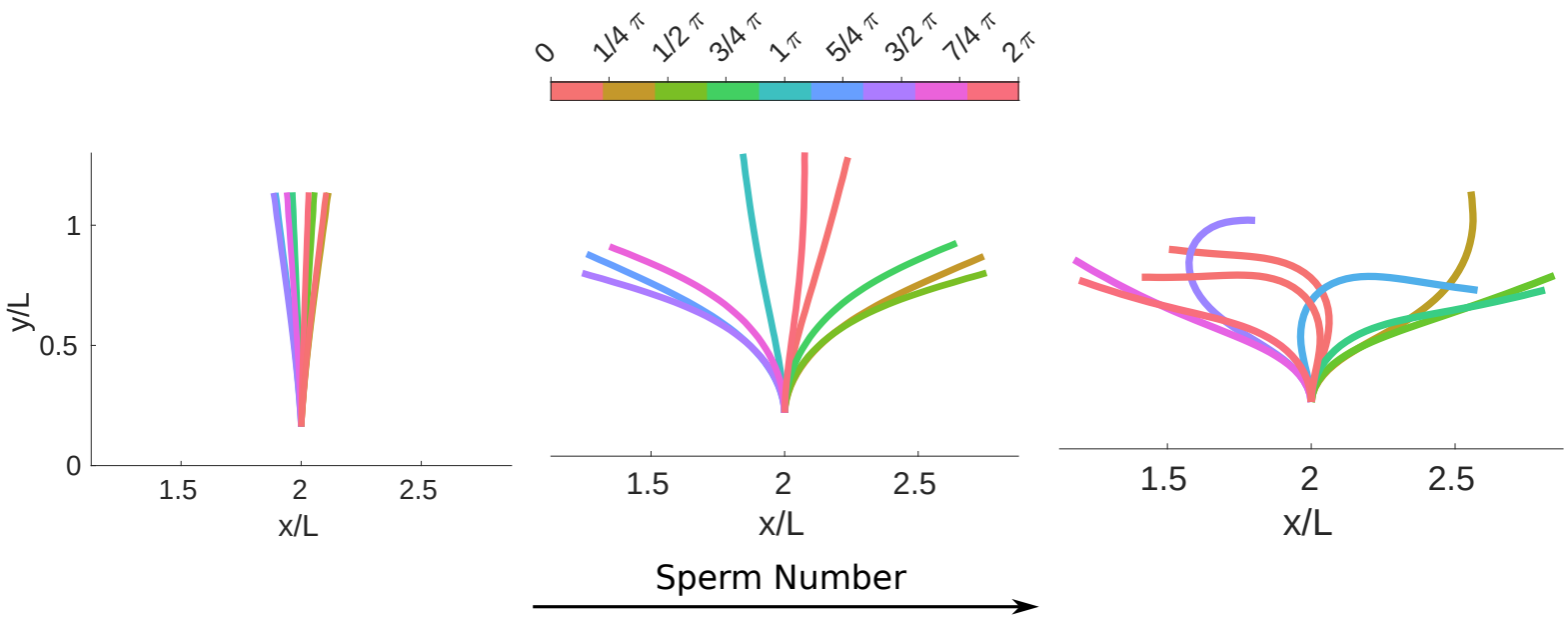
y component of cilia deflection

Data Generation

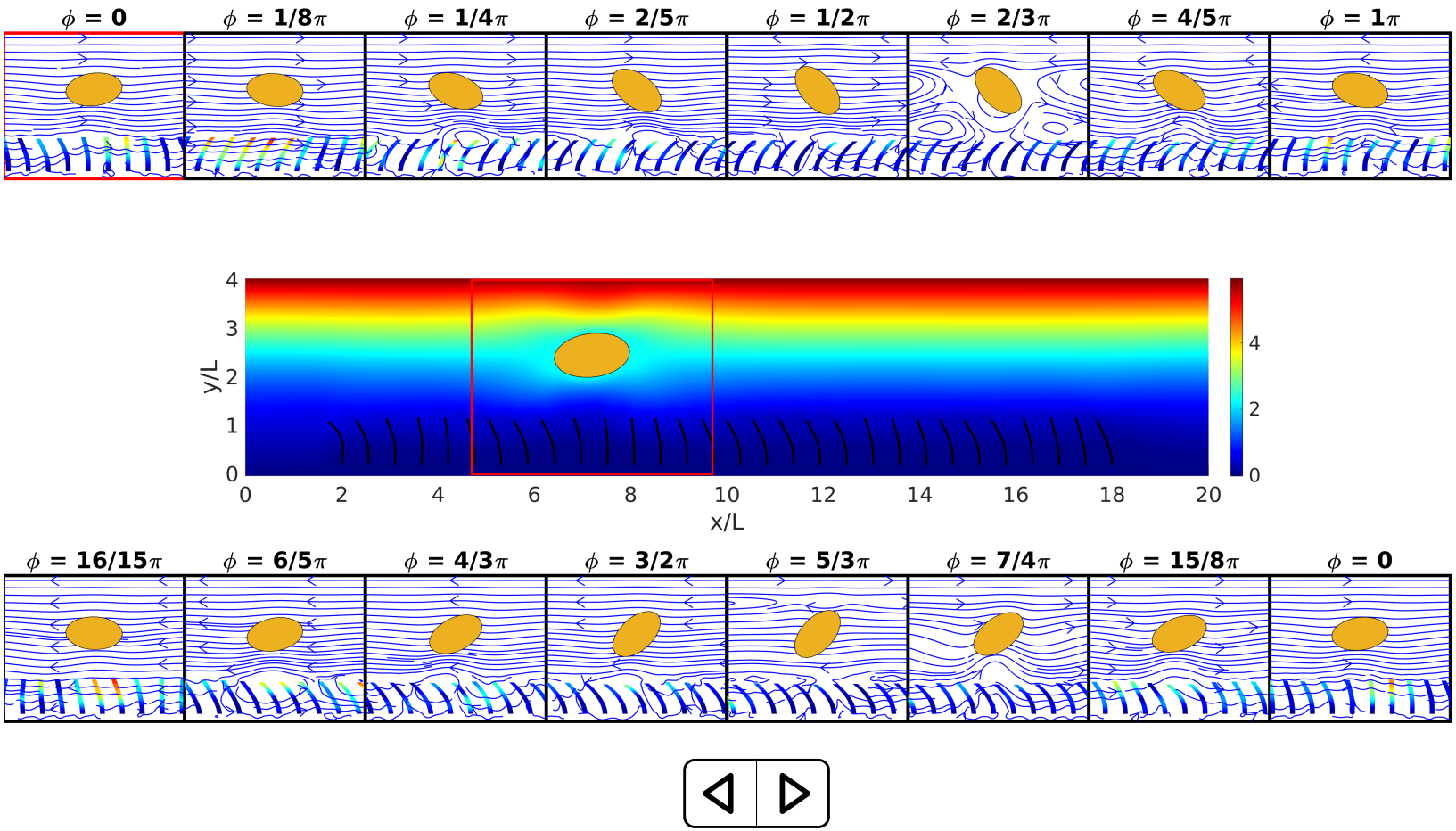


Simulation Parameters

1. Sperm Number , $Sp = L(4\pi\mu\omega/EI)^{1/4} = 0.36$
2. Reynolds Number, $Re = LA\omega/\nu = 5.86$

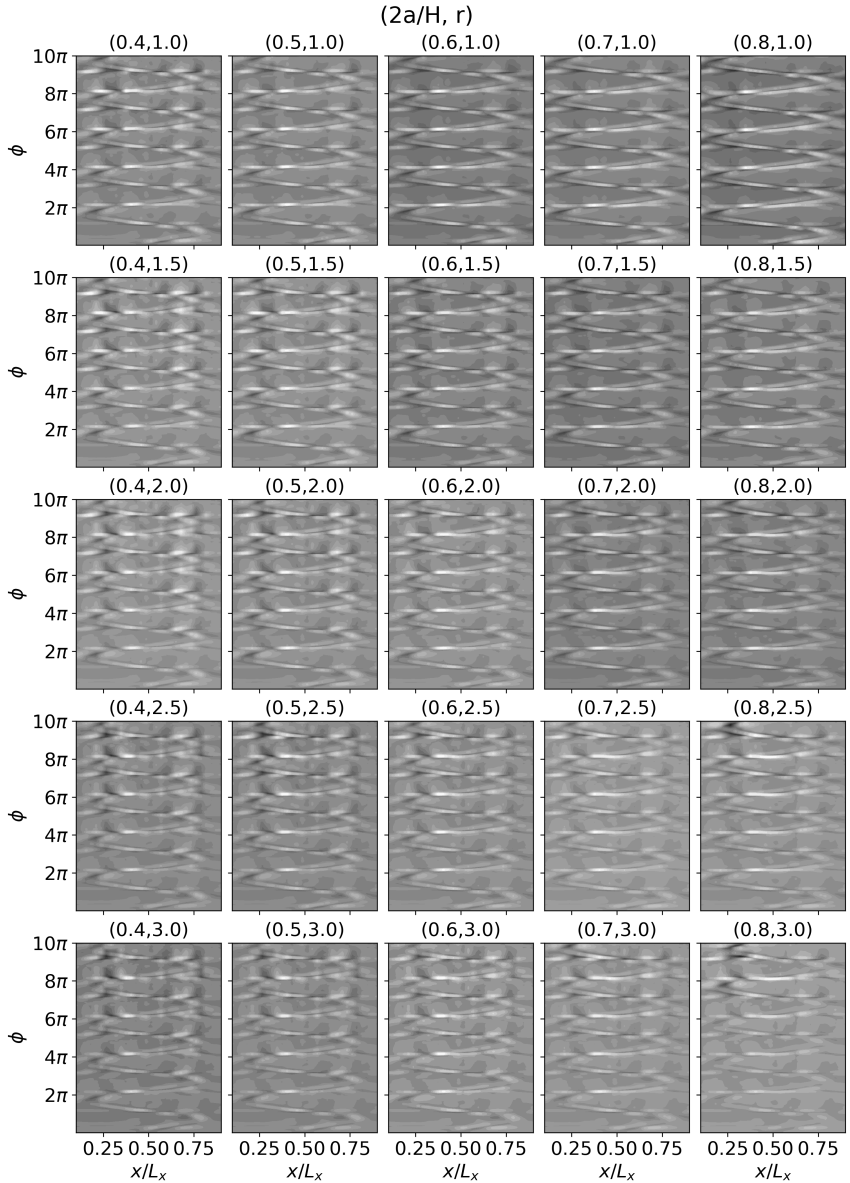


Simulation Results

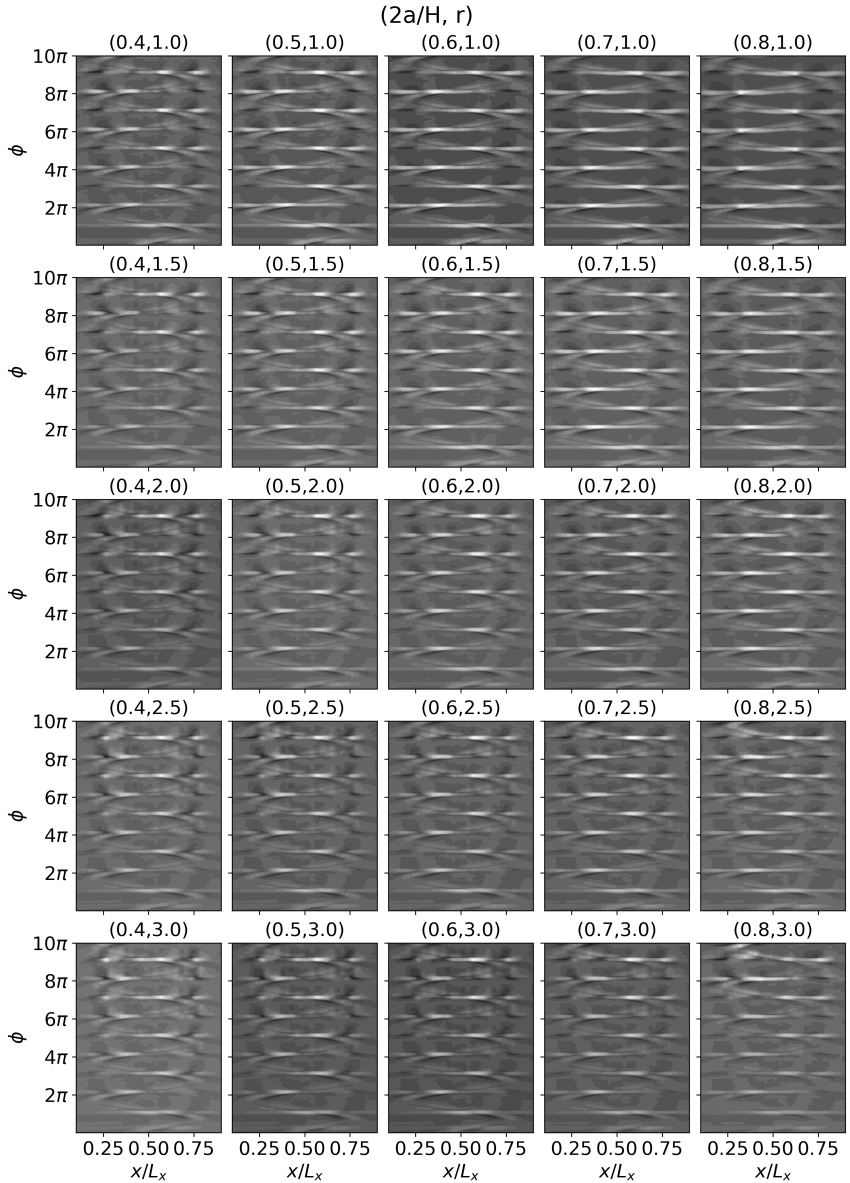


Machine Learning

Data Pre-processing



x-component of fluctuation



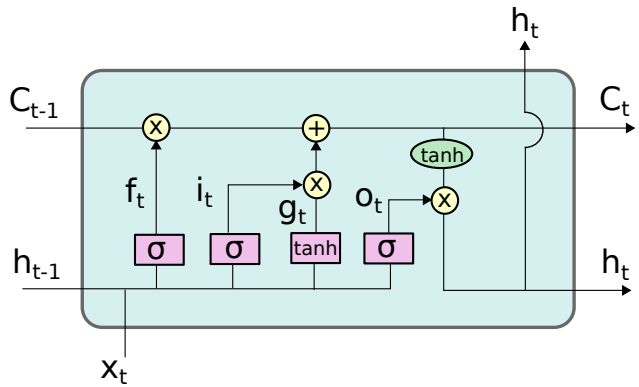
y-component of fluctuation

Training

Each simulation has **750 time instants**. Splitting the data into sequences of 3 we get **250 inputs per simulation** (per pair of (a, r)).

Time Instants	Cilia Deflection Perturbations								Size /Shape
	$x^{1,1}$	$x^{2,1}$	...	$x^{N_c,1}$	$y^{1,1}$	$y^{2,1}$	...	$y^{N_c,1}$	
$1n\Delta t$									
$2n\Delta t$									
$3n\Delta t$									
...									
t_{sim}									

Input data for ML



Long Short Term Memory Network (LSTM) Cell

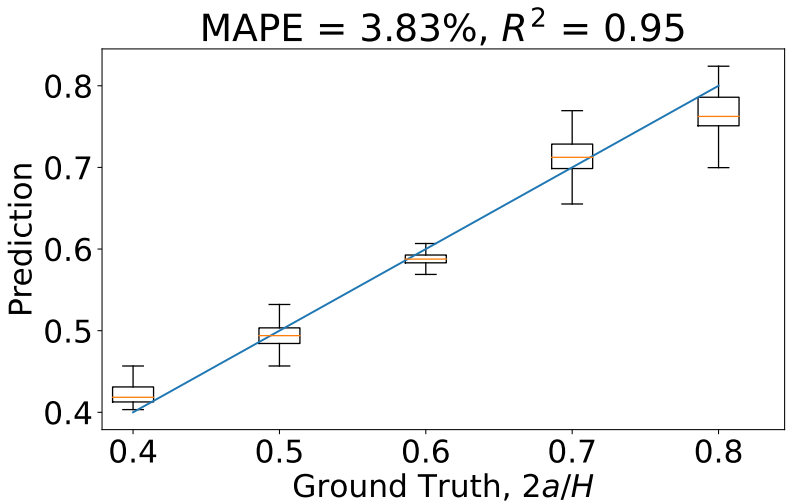
Two models

1. M_t^r: Predicting the particle's aspect ratio using cilia tip deflection perturbations
2. M_t^a: Predicts the particle's semi-major axis using cilia tip deflection perturbations

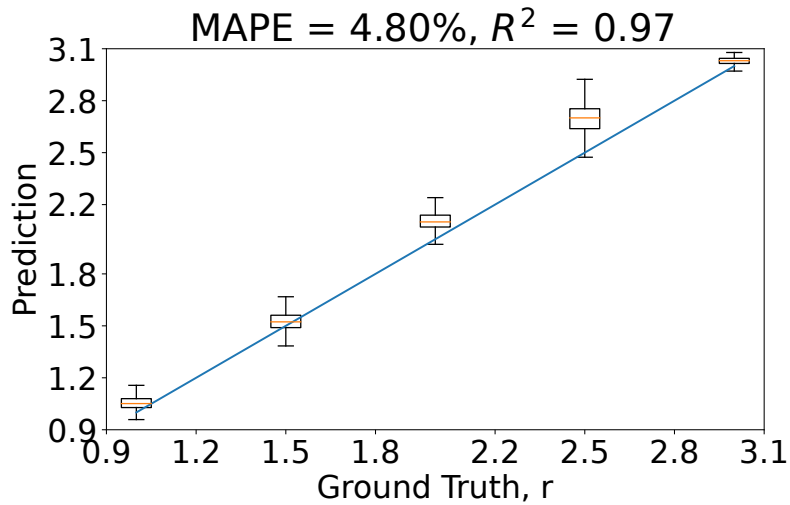
Predictions

$$\text{APE} = \left| \frac{P_p - P_g}{P_g} \right| \times 100$$

$$\text{MAPE}(P) = \frac{1}{250} \sum_{i=1}^{250} \left| \frac{P_{p,i} - P_{g,i}}{P_{g,i}} \right| \times 100$$

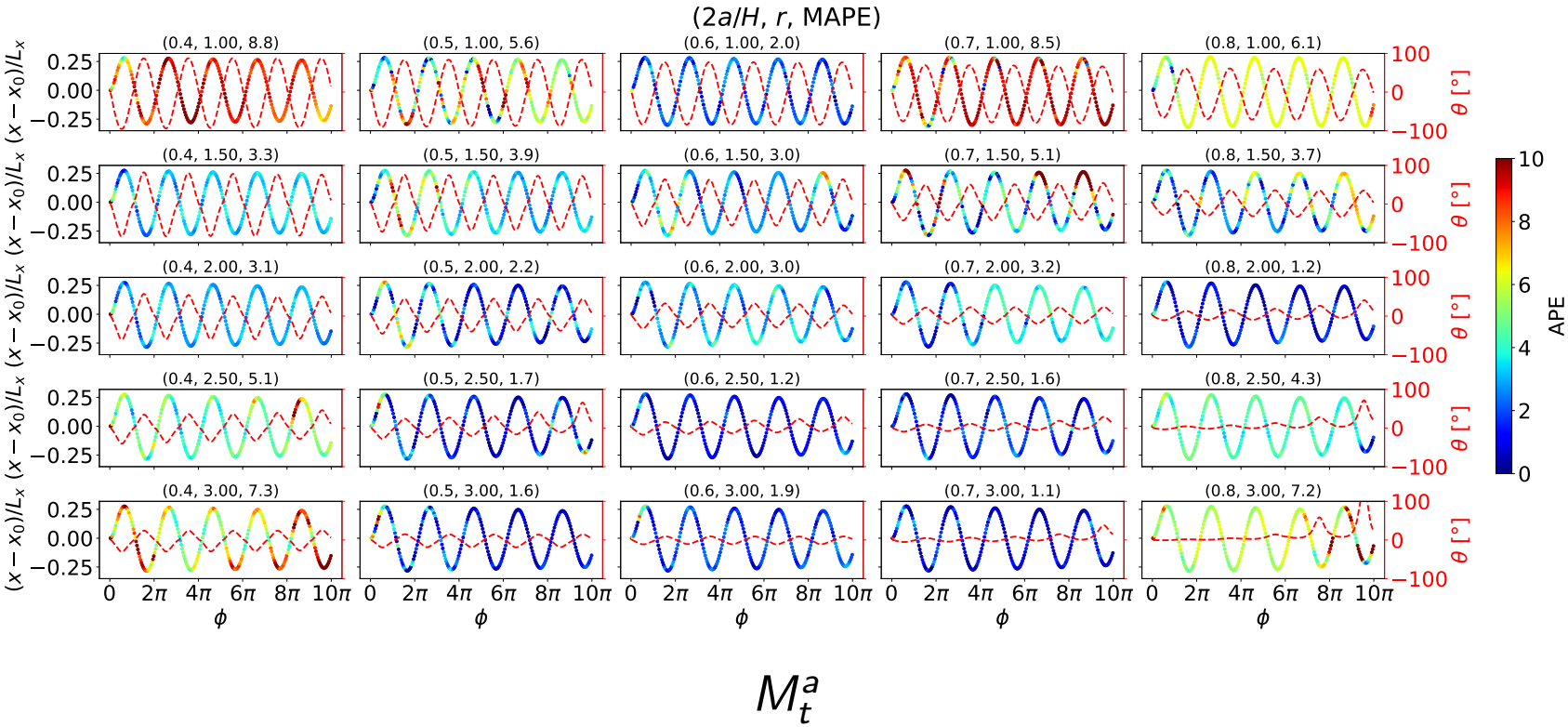


M_t^a



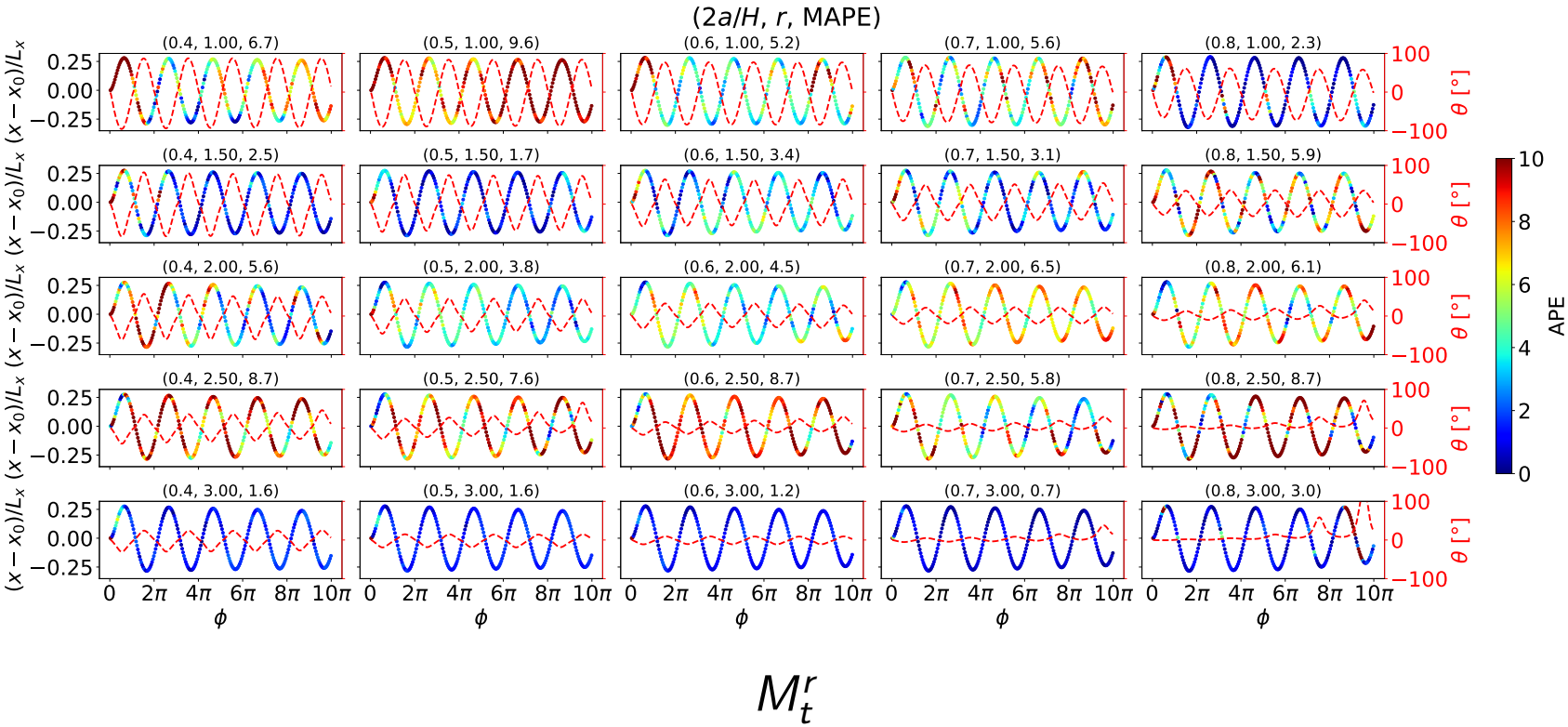
M_t^r

Effect of Parameters on Predictions

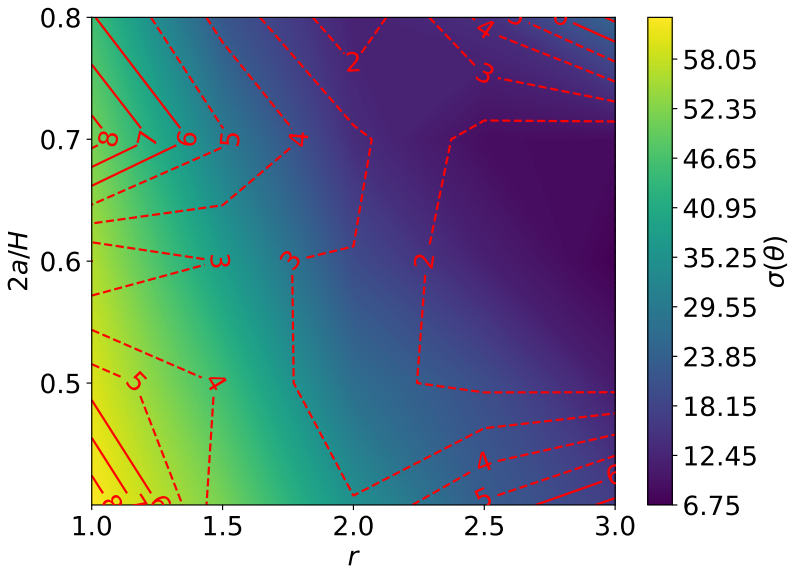


M_t^a

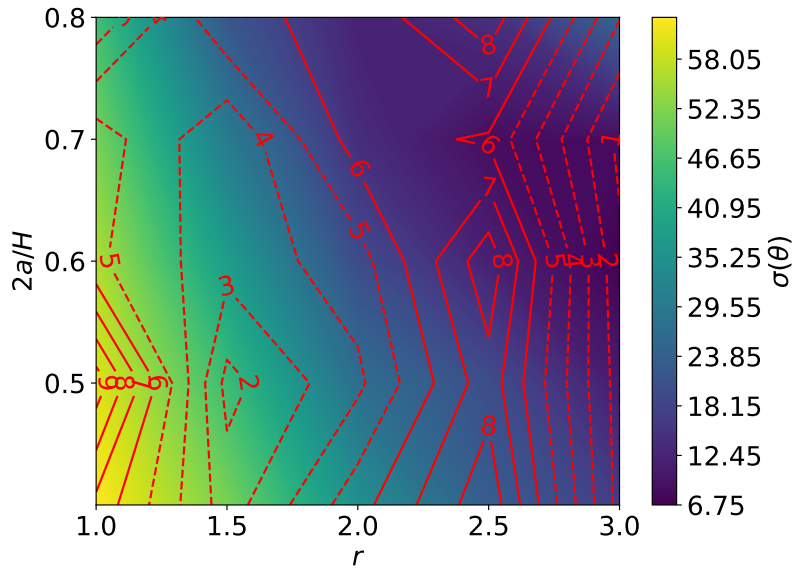
Effect of Parameters on Predictions



Effect of Parameters on Predictions



M_t^a



M_t^r

Summary

- Passive cilia in biological systems act as sensors.
- To mimic this behaviour we simulate cilia-particle interactions and use the data to train machine learning algorithms.
- The trained ML models generate satisfactory predictions.
- Prediction accuracy depends significantly on changes in the particle's orientation as it passes over the cilia array.

References I


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